The Bayesian Brain: Ideal observer models for perceptual decisions (Part 1)

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We need theories to constrain our hypothesis space!

How do we develop useful theories?
Roadmap

Ideal observer modeling:
Uncertainty as guiding principle
in AI and computational neuroscience

(Behavioral) evidence for handling uncertainty

Example: ideal observer models for the speed/accuracy trade-off in perceptual decision-making
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Guiding principle: information is uncertain
Uncertainty handling in artificial intelligence
(a few examples)

Boltzmann machines (stochastic Hopfield networks; Hinton & Sejnowski, 1983)

Bayesian networks (Pearl, 1985)

Statistical learning theory (Vapnik & Chervonenkis, 1971)
- brought us Support Vector Machines (Cortes & Vapnik, 1995)

Variational Bayes; MCMC; ...

... 

Deep learning (~2012): initially no uncertainty

... 

Variational autoencoders (Kingma & Welling, 2014; Rezende et al., 2014)
- build statistical model of inputs

Distributional reinforcement learning (Bellemare et al., 2017)
- build statistical model of long-term rewards
Ideal observer modeling

Brain

AI

Needs to efficiently handle uncertain information

”ideal observer” models

methods for (approximate) inference with uncertain information

potential neural implementations

new algorithmic ideas (e.g., boltzmann machines, networks in general)

my lectures
Principled way of handling uncertainty

Using **Bayesian decision theory** to handle uncertainty

\[
p(\text{state of world} | \text{sensory evidence}) \propto p(\text{sensory evidence} | \text{state of world}) p(\text{state of world})
\]

\[
\text{loss} \begin{pmatrix} \text{true state} \\ \text{assumed state} \end{pmatrix}
\]

**Pierre-Simon Laplace** (*Théorie analytique des probabilités*, 1812):

“The most important questions of life are indeed, for the most part, really only problems of probability”

Cox’s theorem: probabilities are the only principled way to handle uncertainty (Cox, 1946)
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in perceptual decision-making
Combining uncertain evidence from multiple sources

e.g. visual/auditory for object localization
visual/vestibular for self-motion
visual/haptic for bar width estimation

Cue combination using the laws of probability

(Ernst & Banks, 2002)
Rely on prior information

Prior = state of the world in absence of evidence

Real-world: underestimating speed in bad weather

https://www.youtube.com/watch?v=g_sn0WtHK1g
Sensitivity to rewards/losses

(Trommerhäuser, Maloney & Landy, 2008)

...has also been used to reverse-engineer the reward/loss function
e.g. Körding & Wolpert (2004); Drugowitsch et al. (2012)
Recap: Bayesian decision theory

\[ p \left( \text{state of world} \mid \text{sensory evidence} \right) \propto p \left( \text{sensory evidence} \mid \text{state of world} \right) p \left( \text{state of world} \right) \]

- **Posterior**
- **Likelihood**
- **Prior**

Loss function:

\[ \text{loss} \left( \text{true state}, \text{assumed state} \right) \]

Decision:

Rev. Thomas Bayes (1701-1761)
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Perceptual decision-making & the speed/accuracy trade-off

Accumulate evidence over time
Commit to / execute choice

fast choices  speed/accuracy trade-off  slow choices
inaccurate  accurate
low cost of accumulating evidence (e.g. attention, loss of time)  high cost
In the lab: the random-dot motion task
(e.g., Newsome, Britten, Movshon & Shadlen, 1989; Roitman & Shadlen, 2002)

“respond as quickly and accurately as possible”

“left”?  
“right”?  

51.2% coherence  
12.8% coherence
Formalizing evidence accumulation

Latent state

\[ \mu = \{-\mu_0, \mu_0\} \]

Noisy evidence per \( \delta t \)

\[ \delta x_n | \mu \sim N(\mu \delta t, \sigma^2 \delta t) \]

Optimal evidence accumulation: Bayes’ rule

\[
p(\mu | \delta x_{1:N}) \propto p(\delta x_{1:N} | \mu) p(\mu)
\]

\[
g \equiv p(\mu = \mu_0 | \delta x_{1:N}) = \frac{1}{1 + e^{-\frac{2\mu_0 x(t)}{\sigma^2}}}
\]

\[ x(t) = \sum_{n=1}^{N} \delta x_n \]

accumulated evidence
Evidence accumulation by diffusion

\[ \delta x_n | \mu \sim N(\mu \delta t, \sigma^2 \delta t) \]

\[ x(t) = \sum_{n=1}^{N} \delta x_n \]

\[ \frac{dx}{dt} = \mu + \sigma \eta(t) \]

make choices directly in space of accumulated evidence

choose "right"  
\[ \mu = -\mu_0 \]

choose "left"  
\[ \mu = \mu_0 \]
Diffusion decision models (DDMs)
(Ratcliff, 1978)

Works surprisingly well for, fast (<1.5s), single-stage decisions, e.g.,

Word/non-word judgments (e.g., Ratcliff & Gomez, 2004)
Numerosity judgments (e.g., Ratcliff & McKoon, 2018)
Recognition memory (e.g., Ratcliff, 1978)

\[ \frac{dx}{dt} = \frac{\text{drift } \mu}{k c} + \frac{\text{diffusion } \sigma \eta(t)}{c} \]

“coherence” white noise process

(Palmer, Huk & Shadlen, 2005)
Deciding when to decide: decision boundaries

Free evidence: accumulate forever!
Assume: time/evidence is costly

- **cost**: linear in time, $ct$
- **reward**: 1 for correct, 0 for incorrect

**Speed/accuracy trade-off**

- **Fast choices**
- **Slow choices**

- **Cheap**: inaccurate
- **Expensive**: accurate

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**Optimal trade-off**: dynamic programming (Bellman, 1960s)

After accumulating for some time $t$: expected "return" $V(g(t))$

Choosing $-\mu_0$ or $\mu_0$

$1 - g$ or $g$

Accumulating another $\delta t$

$\langle V(g + \delta g) \rangle - c\delta t$

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**Bellman’s equation**

$$V(g) = \max\{1 - g, g, \langle V(g + \delta g) \rangle - c\delta t\}$$
Summary

Ideal observer modeling:
Uncertainty as guiding principle
in AI and computational neuroscience

*Handling uncertainty by Bayesian decision theory*

(Behavioral) evidence for handling uncertainty

*Reliability-weighted cue combination*
*Use of prior information for uncertain evidence*
*Loss-sensitive decision-making*

Example: ideal observer models for the speed/accuracy trade-off in perceptual decision-making

*Optimal speed-accuracy trade-off by diffusion models*

Further questions?