We need theories to constrain our hypothesis space!

How do we develop useful theories?
Roadmap

Ideal observer modeling:
Uncertainty as guiding principle
in AI and computational neuroscience

(Behavioral) evidence for handling uncertainty

Example 1: ideal observer models for the speed/accuracy trade-off in perceptual decision-making

Example 2: Inference on a difference time-scale:
Decision confidence to improve decision strategies
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Guiding principle: information is uncertain
Uncertainty handling in artificial intelligence
(a few examples)

Boltzmann machines (stochastic Hopfield networks; Hinton & Sejnowski, 1983)

Bayesian networks (Pearl, 1985)

Statistical learning theory (Vapnik & Chervonenkis, 1971)
- brought us Support Vector Machines (Cortes & Vapnik, 1995)

Variational Bayes; MCMC; …

Variational autoencoders (Kingma & Welling, 2014; Rezende et al., 2014)
- build statistical model of inputs

Distributional reinforcement learning (Bellemare et al., 2017)
- build statistical model of long-term rewards

Diffusion models, e.g., Stable Diffusion (Sohl-Dickstein et al., 2015; Ho et al., 2020)
- image generation by learning to revers stochastic diffusion process
today's lecture

Brain

AI

Needs to efficiently handle uncertain information

"ideal observer" models

Methods for (approximate) inference with uncertain information

Potential neural implementations

New algorithmic ideas (e.g., boltzmann machines, networks in general)
Principled way of handling uncertainty

Using **Bayesian decision theory** to handle uncertainty

\[ p\left( \text{state of world} \mid \text{sensory evidence} \right) \propto p\left( \text{sensory evidence} \mid \text{state of world} \right) p\left( \text{state of world} \right) \]

**Pierre-Simon Laplace** (*Théorie analytique des probabilités, 1812*):

“The most important questions of life are indeed, for the most part, really only problems of probability”

Cox’s theorem: probabilities are the only principled way to handle uncertainty (Cox, 1946)
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Combining uncertain evidence from multiple sources

e.g. visual/auditory for object localization
visual/vestibular for self-motion
visual/haptic for bar width estimation

Cue combination using the laws of probability

(Ernst & Banks, 2002)
Rely on prior information

Prior = state of the world in absence of evidence

https://www.youtube.com/watch?v=g_sn0WtHK1g

Real-world: underestimating speed in bad weather
Sensitivity to rewards/losses

(Trommerhäuser, Maloney & Landy, 2008)

...has also been used to reverse-engineer the reward/loss function

e.g. Körding & Wolpert (2004); Drugowitsch et al. (2012)
Recap: Bayesian decision theory

$p \left( \text{state of world} \mid \text{sensory evidence} \right) \propto p \left( \text{sensory evidence} \mid \text{state of world} \right) p \left( \text{state of world} \right)$

$p$ (posterior) $\propto$ $p$ (likelihood) $p$ (prior)

$\text{loss} \left( \text{true state}, \text{assumed state} \right)$

$\text{decision}$

Rev. Thomas Bayes (1701-1761)
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Perceptual decision-making & the speed/accuracy trade-off

Accumulate evidence over time → Commit to / execute choice

**fast choices** — speed/accuracy trade-off — **slow choices**

- inaccurate
- accurate
- low cost of accumulating evidence (e.g. attention, loss of time)
- high cost
In the lab: the random-dot motion task
(e.g., Newsome, Britten, Movshon & Shadlen, 1989; Roitman & Shadlen, 2002)

“respond as quickly and accurately as possible”

“left”?  “right”?

51.2% coherence

12.8% coherence
Formalizing evidence accumulation

Latent state

\[ \mu = \{-\mu_0, \mu_0\} \]

Noisy evidence per \( \delta t \)

\[ \delta x_n \mid \mu \sim N(\mu \delta t, \sigma^2 \delta t) \]

Optimal evidence accumulation: Bayes’ rule

\[
\frac{p(\mu \mid \delta x_{1:N})}{p(\delta x_{1:N} \mid \mu)} \propto p(\delta x_{1:N} \mid \mu) \cdot p(\mu)
\]

Posterior belief about motion being “right-ward” (\( \mu > 0 \))

\[
g \equiv p(\mu = \mu_0 \mid \delta x_{1:N}) = \frac{1}{1 + e^{-\frac{2\mu_0 x(t)}{\sigma^2}}} \]

\[ x(t) = \sum_{n=1}^{N} \delta x_n \]

Drugowitsch et al. (2012)
Evidence accumulation by diffusion

\[ \delta x_n \mid \mu \sim N(\mu \delta t, \sigma^2 \delta t) \]

\[ x(t) = \sum_{n=1}^{N} \delta x_n \]

\[ \frac{dx}{dt} = \mu + \sigma \eta(t) \]

Frame \( n \): \( \delta x_n \)

“left”? \( \mu = -\mu_0 \)

“right”? \( \mu = \mu_0 \)

Make choices directly in space of accumulated evidence.

Choose “right”\( \Rightarrow \) Choose “left”\( \Rightarrow \)

Choose “right”\( \Rightarrow \)

Choose “left”\( \Rightarrow \)
Diffusion decision models (DDMs)
(Ratcliff, 1978)

\[
\frac{dx}{dt} = \frac{\text{drift } \mu}{kc} + \frac{\text{diffusion } \sigma \eta(t)}{\text{coherence } t} \]

Works surprisingly well for, fast (<1.5s), single-stage decisions, e.g.,

Word/non-word judgments (e.g., Ratcliff & Gomez, 2004)
Numerosity judgments (e.g., Ratcliff & McKoon, 2018)
Recognition memory (e.g., Ratcliff, 1978)

…
Deciding when to decide: decision boundaries

Free evidence: accumulate forever!
Assume: time/evidence is costly

<table>
<thead>
<tr>
<th>cost</th>
<th>linear in time, ( ct )</th>
</tr>
</thead>
<tbody>
<tr>
<td>reward</td>
<td>1 for correct, 0 for incorrect</td>
</tr>
</tbody>
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**speed/accuracy trade-off**

fast choices \[\leftarrow\] slow choices

cheap \[\rightarrow\] expensive
inaccurate \[\rightarrow\] accurate

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**Optimal trade-off**: dynamic programming (Bellman, 1960s)

After accumulating for some time \( t \): expected "return" \( V(g(t)) \)
(recall, \( g(t) \equiv p(\mu = \mu_0 | \delta x_{1:t}) \))

Choosing \(-\mu_0\) or \( \mu_0 \)

\( 1 - g \) or \( g \)

Accumulating another \( \delta t \)

\( \langle V(g + \delta g) \rangle - c\delta t \)

Bellman’s equation

\[
V(g) = \max\{1 - g, g, \langle V(g + \delta g) \rangle - c\delta t \}
\]
DDMs & Sequential probability ratio test

**Diffusion** Bayes-optimal evidence accumulation

**Decision-boundaries** Optimal speed-accuracy trade-off

**Sequential probability ratio test** (Turning, 1940s; Wald & Wolfowitz, 1948)

Accumulate → Decide

\[ p_H = \prod_{k=1}^{t} p_i(x_k). \]

If

\[ \frac{p_{1n}}{p_{0n}} \geq A^*, \]  the hypothesis \( H_1 \) is accepted,

\[ \frac{p_{1n}}{p_{0n}} \leq B^* \]  the hypothesis \( H_0 \) is accepted.
Neural correlates of evidence accumulation

Gold & Shadlen (2007);
LIP data from Roitman & Shadlen (2002);
MT data from Britten (1992)

Wurtz (2015)
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Extending diffusion model to higher-dimensional inputs

Maximize reward rate for fixed, known difficulty (e.g., fixed coherence) 
(Turing, 1941; Wald & Wolfowitz, 1948)

difficulty that varies across trials 
(Drugowitsch et al., 2012)

difficulty that varies within trials, multiple sources of information 
(Drugowitsch et al., 2014; Drugowitsch, Moreno Bote & Pouget, 2014)

usually
only 2 inputs
known input weights

more realistically
input is larger neural population
inputs weights need to be learned
A larger input population

Learning input weights after feedback $y^*$

\[ p(\mathbf{w} | \mathbf{x}, t, y^*) \propto p(y^* | \mathbf{w}, \mathbf{x}, t) \cdot p(\mathbf{w}) \]

- posterior belief after feedback
- likelihood of weights given feedback
- belief prior to feedback

\[ \mathbf{x}, t \xrightarrow{\mathbf{w}} y^* \]
Approximating Bayes rule

\[ p(w | x, t, y^*) \propto p(y^* | w, x, t) \ p(w) \]

from generative model: cumulative Gaussian likelihood

**Assumed density filtering**

Minimizing \( KL (p||q) \) between true posterior \( p \) and Gaussian approximation \( q \)

\[ q(w | x, t, y^*) \propto p(y^* | w, x, t) \ q(w) \]

assumed Gaussian \hspace{1cm} \text{becomes Gaussian}

Learning of input weights: tracking mean and covariance of Gaussian
The Bayes-(near)-optimal learning rule

With prior belief before feedback, \( \mathbf{w} \sim \mathcal{N}(\mu_w, \beta_w \bar{\Sigma}_w) \)

\[
\Delta \mu_w = \frac{y^* C_w}{\sigma \sqrt{t + \frac{\sigma^2}{\sigma^2_w} + \beta_w^T \bar{\Sigma}_w \mathbf{x}}} 
\]

What determines decision confidence?

- slower choices (usually harder)
- less certain about weights

less confident due to

[Diagram showing decision confidence levels: correct, uncertain, incorrect.]

[Graph showing decision time and weight uncertainty.]
The Bayes-(near)-optimal learning rule

With prior belief before feedback, \( \mathbf{w} \sim \mathcal{N}(\mu_w, \beta_w \Sigma_w) \)

\[
\Delta \mu_w = \frac{y^* C_w}{\sigma \sqrt{t + \frac{\sigma^2}{\sigma^2_w} + \beta_w \frac{x^T \Sigma_w x}{\sigma^2}}} \beta_w \Sigma_w x
\]

- Monotonic function of decision confidence
- Accumulated evidence
- Decision time
- Learning rate \( \alpha_{ADF} \)
- Weight uncertainty

Decision confidence:
- Incorrect
- Correct

Decision time [s]:
- Incorrect
- Correct

Uncertainty about \( w \):
- Certain
- Uncertain
How good is the approximation?

Compare performance to optimal (Gibbs sampling) solution

- **2 inputs**
- **5 inputs**
- **10 inputs**

**ADF**

- **Gibbs sampling**
Do we need to be probabilistic? Simpler heuristics

**Simple delta rule**
minimizes distance between chosen and correct bound

\[ \Delta w = \alpha \left( y^* \theta - w^T x(t) \right) x(t) \]

either \( 2\theta \) or \( 0 \)

**Normalized delta rule**
delta rule with normalization,

\[ w \leftarrow \frac{w}{\|w\|} \langle \|w^*\| \rangle \]

**Stochastic gradient ascent**
on log-likelihood

\[ \Delta w = \alpha \nabla_w \ln p(y^* | w, x(t), t) \]
Steady-state performance

So far

Stationary weights aim to learn weights

Now

Weights follow AR(1) process aim to track changes
Continual learning predicts sequential choice dependencies

- Continual learning models make repeating the same choice more likely based on decision confidence.
- A graph shows the relationship between decision confidence ($C_w$) and repetition bias, with incorrect choices having lower confidence and correct choices having higher confidence.
- The graph also illustrates the probability of choosing right given a previous correct choice, with the probability decreasing as the decision confidence increases.

The figure demonstrates how previous choices and decision difficulties influence current choices, supporting the win-stay and lose-shift strategies in sequential decision-making tasks.
Odor categorization/identification task
Task conditions

Odor identification condition

Odor categorization condition
Vanilla diffusion models can’t fit both conditions
...but a learning model can
Sequential effects
Sequential effects are not fitted, but predicted

Odor identification condition

Odor categorization condition
Simpler models fit data less well

Model comparison on psychometric/chronometric curves & sequential effects

Fitting everything (incl. sequential effects)

Fitting only psychometric/chronometric curves

Using BIC; qualitatively same results for AIC & AICc
Summary

Ideal observer modeling:
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*Handling uncertainty by Bayesian decision theory*

(Behavioral) evidence for handling uncertainty

*Reliability-weighted cue combination*
*Use of prior information for uncertain evidence*
*Loss-sensitive decision-making*

Example 1: ideal observer models for the speed/accuracy trade-off
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*Optimal speed-accuracy trade-off by diffusion models*

Example 2: Inference on a difference time-scale:
Decision confidence to improve decision strategies

*Bayes-optimal learning is confidence-weighted*
*Provides computational role for decision confidence*
*Predicts sequential choice dependencies for continual learning*

Drugowitsch et al. (2012). The Journal of Neuroscience
Drugowitsch et al. (2019). PNAS
Mendonça et al. (2020). Nature Communications
Ideal observer models as hypothesis generators

Brain is too high-dimensional to fully explore by experiments

Use of ideal observer models to generate hypotheses of brain function
Further questions?
Do rodents learn both weights and biases?
Simple parameter adjustment across blocked conditions?
Confidence-weighting only for diffusion models?

For diffusion models

\[ p(w|x, t, y^*) \propto p(y^*|w, x, t) \ p(w) \]

More generally, following choice \( y \) after observing \( x \)

after correct choices, \( y = y^* \)

\[ p(w|x, \text{choice}) \propto p(y = \text{choice}|w, x, \text{choice}) \ p(w) \]

after incorrect choices, \( y \neq y^* \)

\[ p(w|x, \text{choice}) \propto (1 - p(y = \text{choice}|w, x, \text{choice})) \ p(w) \]

What about \( N\)-AFC with \( N>2 \)?

Feedback is correct/incorrect: again decision confidence

Feedback is correct choice: need full posterior \( p(y|w, x) \) for all \( y \)
Confidence trumps imperfect feedback

\[ \beta = \text{probability of inverted feedback} \]
Further questions?